

# Shared Authority and the Economic Organization of Financial Positions: A Theory of Relationship Portability

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## Abstract

Tokenized financial assets can be natively transferable and programmable, but the relationships around them remain commonly coordinated through provider-controlled accounts. This paper studies when a position can persist as the common object of accepted claims and service mandates as its uses or operating providers change. We define **shared authority**: bounded powers held by separately identified actors over a continuing position, with relevant state transitions coupled to accepted commitments. Technical continuity becomes institutional portability within a declared domain only when a qualified outsider recognizes that state and changes an economic decision without requiring the relationship to be reconstructed. The paper establishes three consequences of the mechanism. First, in a sequential financing economy with finite dated resources, we bound owner value across all policies subject to a specified repayment-source restriction and construct a fully funded arrangement that strictly exceeds the bound under stated conditions. Second, we characterize compatible specialist entry, voluntary formation and transferable service value around inherited claims. Third, a stock-lending application connects advance acceptance to executable opportunities and derives quantities, rents and valuation in an established equilibrium. The results identify when authority design expands the useful financial relationships supported by existing capital and how the resulting benefits are divided.

**Keywords:** Shared authority; relationship portability; financial contracting; capital coordination; creditor protection; securities lending; financial composability; modular brokerage.

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## 1. Introduction

Digitization made financial records electronic. Tokenization can make asset state natively transferable and programmable. Neither change by itself makes the financing, protection, priority or remedies surrounding an asset portable across providers. Many capital-market relationships remain organized through provider-controlled accounts: a position receives financing, execution or servicing only after entering a provider’s contractual, recordkeeping and control perimeter. Moving the asset does not automatically carry the accepted commitments on which those relationships depend.

Capital markets therefore do more than transfer assets. They organize continuing relationships among owners, creditors, custodians, trading venues, protection sellers and service providers. A position can support several of these relationships only if each party can rely on the claims, priorities and remedies it has accepted. The allocation of authority is a design constraint on both financial products and financial organization: it determines who may act, whose consent is required, and which commitments must survive an action. When can the position, rather than the provider-controlled account, become the continuing unit around which these relationships are organized?

Integrated accounts offer one answer. A broker, bank or custodian can coordinate financing, execution and servicing inside an institutional perimeter because its contracts, records and controls govern the position. Traditional markets also coordinate across institutions, but doing so requires accepted legal, operational and balance-sheet arrangements. Integration is consequently valuable for reasons that include netting, capital, judgment, loss absorption and convenience. It can also make access to additional services depend on the provider that controls the account, even when those services do not require general control of the position.

Programmable asset state changes this organizational problem. An on-chain position can participate in automated and contingent actions supplied by different applications, expanding possible combinations of financing, protection and use (BIS, 2023). Yet technical programmability does not make those actions financially compatible. A transfer permitted by one application may remove collateral or delivery capacity on which another party relies; revoking an operator may strand a payment or remedy; two locally valid permissions may promise the same position to incompatible uses. As service provision becomes easier to distribute, authority over the resulting commitments becomes more important.

We study **shared authority**: bounded powers exercised by separately identified actors over a common financial position under enforceable rules. A permission specifies what an operator may request; a commitment specifies another party’s entitlement. Write-domain separation distinguishes authority over accepted commitments from authority to request a service, while asset-authorization coupling requires relevant transitions to respect the live commitments. A continuing position carries the accepted beneficiaries, priorities, dates and remedies through permitted changes in use and specified changes of operator.

The hierarchy is important. Shared authority is the mechanism; the continuing position is the institutional object; governed succession is the process by which an operating role changes; recognition is the condition under which another actor relies on the resulting state; and relationship portability is the outcome within that recognition domain. Portability here is not the movement of a token alone. It is the continuation of specified financial relationships without reconstructing the position and its inherited terms inside every provider’s private perimeter.

Consider a financed share. The lender relies on repayment priority and control of the position inside an accepted account arrangement. A specialist later offers securities lending, protection or option

execution, but its instruction may remove the backing or delivery capacity supporting the lender. Requiring the investor to close the loan, move the share or post another complete pool preserves the lender but reconstructs the relationship. Shared authority instead admits the specialist only under a bounded mandate that preserves the inherited priority and funds any required substitution, delivery or remedy. The share and its issuer cash flows need not change; what changes is which later service can act and under what conditions. Whether that change creates value remains an economic question rather than a consequence of programmability alone.

This arrangement defines a form of **position-centered financial organization**. Ownership and accounting, authority to act, coordinated execution, and risk or liquidity provision are separable design choices; none is determined by the label “pool,” “account” or “protocol.” Integrated provision remains one admissible organization. Shared authority adds another: independent specialists may receive narrow service mandates around the same position while inherited claims continue to bind. We call a position **economically programmable** within a declared domain when its uses and service relationships can change under accepted rules without invalidating relied-upon commitments or leaving required performance unfunded. The design space has both a simultaneous dimension, which services can coexist, and a temporal dimension, which commitments survive valid state and provider changes.

The paper develops the economic theory of this organization. Its results address three margins. The first is **protection and use**: a rule that protects one claim can exclude a productive use of the same capital, imposing a control tax beyond any explicit intermediary fee. The second is **provision and formation**: accepted rules can permit an additional specialist to serve an already financed position, but only if the new value covers actual costs and the affected parties accept the distribution of gains. The third is **value and incidence**: making an opportunity executable can change activity and the value of a transferable entitlement, while competition or greater execution can move rents away from the original owner. These margins distinguish feasible relationships, resource productivity and future service choice without treating them as a single cost statistic.

Section 3 gives the paper’s policy-wide quantitative result. In a sequential financing economy, choosing a more productive later activity makes its entire receipt unavailable to original creditors. Forgone production and creditor risk impose an upper bound on owner value across all admissible policies, including qualifying pooling, guarantees, refinancing and fractional allocations. A fully funded coordinated arrangement exceeds the bound under stated conditions with unchanged technologies and initial resources. Appendix I specifies a sufficient contracting domain. This is one exact instance of the broader protection/use problem; the repayment-source restriction is a declared premise, not a general consequence of provider separation.

Sections 4 and 5 study the other margins in separate economies. Section 4 gives necessary and sufficient conditions for additive specialist entry around inherited claims, then analyzes voluntary formation and transferable service value. Section 5 connects an advance-accepted and fully funded stock-lending opportunity to execution, bargaining and price in an established equilibrium, and uses physical share delivery as a lifecycle stress test. The models are complementary rather than additive pieces of one universal welfare calculation. Together they show how authority design can change the useful relationships supported by a position, the organization of their provision and the division of the resulting surplus. This paper takes the feasibility of each declared authority domain as given and studies its economic consequences; it does not derive a general product frontier over alternative technical or institutional architectures.

## 1.1 Contribution in relation to prior work

Control rights and financial access have established foundations. Hart and Moore (1988, pp.6–10) study control through assets available to coalitions; Rajan and Zingales (1997, Propositions 4–5) distinguish access from ownership. Aghion and Tirole (1994, Sections 3.2–3.4) distinguish formal authority from effective control, while their innovation model (1993, pp.20–22, 28–29) combines external finance with separable property and control rights. These theories supply essential primitives for allocating power. Our object is the continuing financial position around which several accepted claims and service mandates may coexist. We ask how bounded powers change its feasible uses, the organization of service provision and the value distributed among participants.

Organizational law supplies a distinct source of continuity. Hansmann and Kraakman (2000, Parts III–IV) analyze affirmative asset partitioning and liquidation protection; Hansmann and Kraakman (2002) connect divided property rights to the verification costs imposed on third parties. Shared authority does not reproduce those legal effects by technical assertion. It may operate beneath a legal entity, recognized contract or other institutional anchor. We hold the relevant recognition perimeter fixed and study what uses and service relationships bounded position-level authority permits inside it.

Restrictions also have established incentive and financing effects. Holmström and Milgrom (1991, Propositions 3–4) analyze permitted activities and incentives; Holmström and Tirole (1994, Sections 3.1–3.2) connect monitoring restrictions to pledgeable income. Leitner (2010) connects coordination under hidden contracts to collateral and investment. Our incremental result is a quantitative upper bound on owner value from this protection/use tradeoff across complete policies, together with a funded arrangement exceeding it. The model uses certified opportunities, concave creditor participation and finite dated resources; it does not solve endogenous monitoring or hidden-action problems.

Private-computation systems supply architectural antecedents. Zexe binds private records to predicates governing their creation and consumption and specifies funded conditional exchanges executable by other parties (Bowe et al., 2020, Sections 2.3 and 6.2). Hauser (2026) specifies a different encumbrance construction: separate spending and encumbrance identifiers, persistent restriction state, and a spend proof coupled to that state, with condition-bound enforcement. Section 5.3 distinguishes these constructions. The present contribution is economic rather than a claim that these enforcement primitives are individually new: it develops position-centered financial organization and derives a quantitative protection/use comparison, conditions for entry around inherited claims, and the valuation and distribution of executable service opportunities.

BIS (2023) discusses programmable claims and expanded contracting possibilities. We focus on the authority conditions under which independent services can rely on compatible commitments throughout a position’s lifecycle. The valuation application builds on Duffie, Gârleanu and Pedersen (2002), who price securities-lending income, and relates to Gârleanu and Pedersen (2011), who connect funding requirements to asset prices. Our contribution there is a funded authorization-to-execution construction and its economic incidence, not a new general pricing principle. Section 3.5 gives the proposition-level comparisons; citations to working papers refer to the versions in the bibliography.

Section 2 defines the mechanism and economy. Section 3 derives the financing result and its boundaries. Section 4 studies entry, formation and transferable value. Section 5 develops the stock application and implementation comparisons. All proofs and their material assumptions remain in this paper. Owner value, stock price, welfare and minimum capital for an identical protected

outcome are distinct objects.

## 2. Position-centered organization and the contracting environment

### 2.1 Continuing positions, accepted commitments and bounded powers

For this paper, a financial position combines an asset or cash-flow interest with the accepted claims, permissions and remedies governing its use. It is **continuing** within a domain when those commitments remain defined through permitted state transitions and specified changes of operating provider. Continuity concerns the position’s accepted financial terms; it does not by itself establish legal portability across systems or replace a creditor, insurer or other financial counterparty.

A position is **technically continuing** when its accepted state survives the specified changes in use or operator. It is **institutionally portable** within recognition domain  $Q$  when an independent qualified actor can verify the relevant state, accept the rights and responsibilities that the governing terms assign to it, and change an economic decision without requiring the inherited position to be closed and reconstructed. Recognition may be contractual, legal, operational or technical and is actor- and effect-specific. None of the results below establishes unrestricted portability across issuers, custodians, counterparties or legal regimes.

Ownership and accounting, authority to request an action, coordinated execution, and risk or liquidity provision are distinct dimensions of the arrangement. A pooled vehicle may use narrow mandates; individually held positions may be subject to joint commitments; an integrated intermediary may implement the same bounded rules internally. Institutional labels therefore do not determine the effective allocation of authority. What matters is which actors can change relevant state, which claims constrain those changes and which resources fund performance and remedies.

An authority constraint binds when an otherwise funded, mutually acceptable arrangement cannot proceed under the available powers and consent rules. Changing the governing commitments can change a position’s financial opportunities without changing the issuer’s underlying cash flows. The shared-authority specification contains an authoritative position state, commitment records, role-specific transitions and the asset/settlement paths those transitions control. Each commitment identifies beneficiary, units, dates, priority, triggers, remedies and agreed discharge or amendment. Operators and beneficiaries are separate identities.

Three conditions are essential. **Acceptance:** affected parties accept the charter before reliance or consent to amendments. **Complete mediation:** every asset-affecting path, including withdrawal and privileged upgrades, obeys its guards. **Persistence:** actions, revocation of future access and operator replacement preserve actual accepted claims and remedies until agreed termination. This protects contractual contingencies, not unconditional payment in every state. Public discovery can help verification; neither publicity nor a particular ledger is necessary or sufficient.

A **write domain** is the state an actor can change through transactions authorized by its key alone. An accepted restriction cannot persist if the owner or an operator can erase it outside its agreed discharge rules, or move the asset through an unchecked path. The design therefore combines separated write authority with **asset-authorization coupling:** validity of each relevant asset transition depends on the live commitment state (Hauser, 2026, Sections 3–4).

The private-state construction binds a note, a condition tree and a redistribution intent. It uses separate, domain-tagged identifiers for ordinary spending and encumbrance: the former depends on the owner key, the latter on note randomness. A spend proof binds the encumbrance identifier to the same note; the registry checks active restrictions. Condition-bound enforcement follows the

committed intent without requiring the owner to co-sign. These properties depend jointly on the proof checks, cryptographic assumptions and fixed registry rules; separate identifiers alone do not establish them. The public register implementation in Section 5.3 realizes claim-preserving authority through a different state representation.

Claim preservation requires protecting the backing as well as the record of an entitlement. Accepted terms must identify the parties responsible for performance and remedies; replacing an operator does not itself transfer their financial obligations. For Theorem 1, Appendix D proves preservation under funded payment partitions and a bounded handoff that changes the operator while preserving beneficiaries and financial terms. Successor availability, physical capacity and timely performance remain separate requirements.

Admitting a specialist, replacing an operator and replacing a financial counterparty are distinct changes. For Proposition 2, Appendix G keeps the added service’s account separate and its operations compatible with inherited execution. Replacing a lender or moving the asset to another custody or enforcement system requires its own accepted terms and compatibility conditions. The constructions establish specified forms of continuity, not unrestricted portability of financial relationships.

Across the paper, an authority regime induces a feasible continuation set  $\mathcal{F}_R(c)$  from accepted commitment state  $c$ . A **compatible expansion** from  $R$  to  $R'$  preserves the inherited claims, backing and remedies represented by  $c$  while adding at least one fully funded continuation:  $\mathcal{F}_R(c) \subsetneq \mathcal{F}_{R'}(c)$ . This ordering is technological and contractual, not a welfare result. An added continuation creates strict economic value only when it produces net surplus, satisfies participant acceptance and preserves any value or entitlement being transferred. Section 3 studies a protection/use expansion, Section 4 an entry and formation expansion, and Section 5 an execution expansion and its incidence. The three economies are separate specializations of this common object.

## 2.2 Economy and admissible financing policies

The first economy isolates one consequence of position-centered authority: protecting an original claim can make a later productive receipt inaccessible even when the later activity is fully funded. Sections 4 and 5 use separate environments for entry and valuation. The distinction prevents the source-restricted financing result from standing in for every form of financial composability.

Table 1: Notation and financial roles.

| Notation        | Meaning                                                                                                                     |
|-----------------|-----------------------------------------------------------------------------------------------------------------------------|
| N, T, S         | Original creditors, later financiers and coordinating services, respectively; these labels do not imply institutional form. |
| $n = 2G$ ; k; m | Number of positions and two-position groups; total initial investment; installed service groups.                            |
| a, v; $y_i$ , g | Initial cash and noncash use per invested unit; base later output and superior-output increment.                            |
| K, F, R, H      | Actual service capital, setup expenditure, dedicated recovery reserve and service participation/carry burden.               |
| $\delta$ ; q    | Conditional expected recovery frequency; lower probability bound on the critical event.                                     |

| Notation             | Meaning                                                                                                           |
|----------------------|-------------------------------------------------------------------------------------------------------------------|
| $y_H, y_L; p_H, p_L$ | High/low type outputs and repayments; H/L subscripts are type labels, whereas standalone H is the service burden. |
| $W; \ell; U$         | Expected best later-only surplus; statewise productive surplus forgone; owner cash plus initial use value.        |

Table 1 summarizes the notation. There are  $n = 2G$  positions. At date 0, original creditors have  $n$  units of actual cash and  $G$  services each have  $K \geq F + R$ . Initial investment  $x_i \in [0, 1]$  costs  $x_i$ , yields terminal cash  $ax_i$  with  $0 \leq a < 1$ , and gives its owner noncash use  $vx_i$ . Write  $k = \sum_i x_i$  and  $d = 1 - a$ . Installing  $m$  service groups requires  $mF$  actual setup work, supports at most  $2m$  initially financed positions, and locks  $R$  per group for the specified one-failure recovery. Thus  $0 \leq k \leq 2m \leq n$ .

At date 1, later financiers supply  $n$  separate actual units. A position's funded base route returns certified  $y_i$ , with  $1 < y_i \leq \bar{y}$ . An available superior route returns  $y_i + g$  with  $g > 0$  from the same input. In the comparison class, choosing that route makes its entire receipt  $y_i + g$  unavailable as a source for original-beneficiary wealth; the exclusion includes the baseline component  $y_i$ , not only the increment  $g$ . This is a declared conflict between productive use and accessible repayment, not a consequence inferred from separate providers. Fractions of the later input may use base, superior or no production. Unused funds remain in the supplying participant's wealth. All financial beneficiaries remain fixed through pooling, claim sales and institutional wrappers. Productive inputs and real work leave the financing network; undeclared supplier credit or other assets are not available.

Every original creditor has increasing concave utility  $u$ , continuous on the bounded nonnegative wealth domain, differentiable at one with  $u'(1) > 0$ , and requires  $\mathbb{E}[u(X_i)] \geq u(1)$ .  $X_i$  includes retained cash and intermediate consumption net of all advances and repayments. Later financiers require aggregate expected terminal-equivalent wealth at least  $n$ . Services require at least  $GK + H\mathbb{E}[m]$ , counting inactive funds. Recovery spending  $J$  obeys  $\mathbb{E}[J \mid \mathcal{F}_0] = \delta Rm$ . Its complete expected burden is  $c = F + \delta R + H$ . Service acceptance in the construction is based on expected wealth; it is not asserted for an arbitrary risk-sensitive service utility.

Initial allocations, installations and selection are measurable at information  $\mathcal{F}_0$ . There is an event  $E$  with  $\Pr(E \mid \mathcal{F}_0) \geq q > 0$  for every allowed initial choice. On  $E$  every superior route is available and every active recovery reserve is needed. Funded commitments persist through this recovery horizon. A signal, alternative service or early-release contract invalidating these conditions needs a different bound.

In the comparison class  $L$ , no credible payment path from superior receipts supplies original-beneficiary wealth, directly or via a guarantee, refinancing, novation, claim sale or balance sheet. All other permitted pooling, conditional pricing, later senior portfolios, internal guarantees from accessible balances, initial selection, default, waiting and lotteries remain allowed. The comparison is not restricted to the construction's prices or command scheduler.

Let  $U$  be total ex-ante owner cash plus initial use value. Let  $W$  be expected best later-only surplus at later capital's reservation return, before initial service installation. The no-initial-installation technology has the specified zero installation charge. Owner value is distinct from total welfare, minimum capital for a fixed protected outcome, and the supremum in the coordinated class.

### 3. A quantified protection–use trade-off

The financing model quantifies a **control tax** in one exact environment: value forgone because protecting an accepted claim excludes an otherwise productive use. Consider a financed position with a later activity producing more output from the same additional input. In the restricted environment, that activity’s entire receipt is unavailable to original creditors. Retaining the less productive activity preserves a repayment source; choosing the better one can leave a shortfall to be covered from other resources or borne by creditors. One institutional interpretation is an original claim enforceable against receipts inside its accepted account or control agreement, while a later receipt is paid to a fixed outside beneficiary and cannot be assigned, reached or credibly forwarded under the available contracts. A common balance sheet, effective assignment or additional recourse would change the premise. Appendix I specifies fixed payees, absent recourse and terminal conditions that realize this boundary. Other authority constraints can take different forms; the theorem establishes the result for this declared class.

The proof follows the resources available to repay the original creditors. On the critical event, a policy can preserve accessible receipts by giving up productive opportunities, leave creditors exposed to a shortfall, or combine the two. Lost production reduces owner value; concave creditor utility can require additional expected wealth to compensate for risk. Summing the dated budgets and imposing creditor participation therefore places an upper bound on owner value. The inventory deliberately grants favorable access to other balances, so this upper bound need not be attainable.

#### 3.1 The bound and its proof

Set  $B_m = GK - m(F + R) \geq 0$ ,  $\alpha = g/\bar{y}$ , and

$$r_u(x) = x - 1 - \frac{u(x) - u(1)}{u'(1)},$$

$$\Psi(h) = \min_{0 \leq z \leq h} \{\alpha(h - z) + nr_u(1 - z/n)\}, \quad 0 \leq h \leq n.$$

**Theorem 1 (Coordination bound and funded comparison).** Every complete feasible policy in  $\mathcal{L}$  satisfies

$$U_L^{\sup} \leq W + \max_{\substack{m \in \{0, \dots, G\} \\ 0 \leq k \leq 2m}} \{k(v - d) - mc - q\Psi((kd - B_m)_+)\}.$$

A funded coordinated policy with full initial investment, all best available later uses, protected original claims, the specified paid recovery and total excess participant compensation  $\pi$  has

$$U_A = W + n(v - d) - Gc - \pi.$$

Whenever this attained value exceeds the displayed upper bound, it exceeds every qualifying comparison policy. The comparison optimum need not be attained. The existence of this coordinated policy must be established separately; it is not supplied by the inequality alone.

The quantity  $h$  is the critical-event repayment deficiency in the optimistic inventory before allowing base production or unused later inputs to supply accessible resources. In the minimization,  $z$  is a lower bound on the remaining creditor shortfall after allowing for forgone productive use. The term

$\alpha(h - z)$  bounds the associated productive surplus forgone, while the tangent gap bounds creditor risk loss. Unconditional participation translates expected tangent loss into additional expected creditor wealth. Minimizing over  $z$  gives a lower bound on the combined penalty, which enters the owner-value bound with the event's probability floor  $q$ . The minimizing decomposition need not be implementable.

**Proof: resources and creditor participation** Concavity implies  $r_u \geq 0$ , convexity,  $r_u(1) = 0$  and that  $r_u$  is nonincreasing on  $[0,1]$ . Unconditional participation gives  $\mathbb{E}[X_i] \geq 1 + \mathbb{E}[r_u(X_i)]$ . It is not replaced by participation conditional on a rare state or an initial lottery outcome.

Sum the complete dated cash budgets and cancel internal transfers. A claim price is an actual payment, not an extra source equal to both price and face value. Principal reinvested is spent again and cannot remain in terminal wealth. Let  $S^*(\omega)$  be the realized best later-only surplus from the same opportunity menu, with  $W = \mathbb{E}[S^*]$ . Let  $\ell(\omega) = S^*(\omega) - S_{\text{policy}}(\omega) \geq 0$  be the statewise productive surplus forgone by the policy, before participant compensation. Later and service participation imply

$$\begin{aligned} U &\leq W + \mathbb{E}[k(v - d) - mc] - \mathbb{E}[\ell + \sum_i (X_i - 1)] \\ &\leq W + \mathbb{E}[k(v - d) - mc] - \mathbb{E}[\ell + \sum_i r_u(X_i)]. \end{aligned}$$

Setup, recovery and carry enter once. No individual later loan repayment floor is imposed.

**Proof: resources available for repayment** On E write  $s_i, b_i, w_i$  for superior, base and unused later fractions, summing to one. Let  $w = \sum_i w_i$  and  $t = \sum_i (b_i + w_i)$ . Actual later inputs are  $n \cdot w$ . After all actual productive inputs, setup and required recovery, grant original capital every otherwise available balance, even balances for which other participants also have claims. Including first and base outputs, its wealth is bounded by

$$\sum_i X_i \leq n - kd + B_m + w + \sum_i y_i b_i \leq n - kd + B_m + \bar{y}t.$$

The first inequality uses the no-effective-superior-source premise. The second has exact nonnegative slack

$$\sum_i (\bar{y} - y_i) b_i + (\bar{y} - 1)w.$$

The actual lost surplus is

$$\ell = \sum_i [gb_i + (y_i + g - 1)w_i] \geq gt.$$

These identities allow all fractions, redistribution, idle balances and internal transfers. Recovery that remains locked cannot simultaneously finance repayment. As a boundary control, suppose superior production preserved an enforceable original-creditor payment from its baseline component  $y_i$ . The accessible-source inventory would then include  $\sum_i y_i s_i$ . The displayed all-superior deficiency would

no longer follow. This defeats this particular certificate; it does not by itself construct a funded alternative or prove equality of attainable values.

**Proof: protection versus productive use** Let  $h = (kd - B_m)_+$ . Because  $k \leq n$ ,  $d \leq 1$  and  $B_m \geq 0$ ,  $h \leq n$ . If  $t < h/\bar{y}$ , the original creditors' mean wealth is at most  $1 - (h - \bar{y}t)/n$ . Jensen's inequality and monotonicity of  $r_u$  below one yield

$$\sum_i r_u(X_i) \geq nr_u \left( 1 - \frac{h - \bar{y}t}{n} \right).$$

Put  $z = h - \bar{y}t$ . Combined loss is at least  $\alpha(h - z) + nr_u(1 - z/n)$ , hence at least  $\Psi(h)$ . If  $t \geq h/\bar{y}$ , productive loss alone is at least  $\alpha h$ , which is the feasible  $z = 0$  value and therefore at least  $\Psi(h)$ . Outside E the combined loss is nonnegative.

The initial  $h$  is  $\mathcal{F}_0$ -measurable and nonnegative. Consequently the conditional probability premise implies  $\mathbb{E}[\mathbf{1}_E \Psi(h) \mid \mathcal{F}_0] \geq q\Psi(h)$ . Substitute into the budget and maximize over all allowed  $k, m$ . Randomized initial policies cannot exceed that maximum. This proves the all-policy upper bound. A feasible coordinated construction has no lost superior use and pays its excess compensation  $\pi$ ; its displayed value follows from the same budget. Comparing the two proves the final implication. QED.

### 3.2 Strictness and a funded construction

$\Psi$  is nondecreasing and  $\alpha$ -Lipschitz. For  $h_2 \geq h_1$ , reusing a minimizer at  $h_1$  proves the upper Lipschitz bound. A candidate  $z \leq h_1$  costs at least  $\Psi(h_1)$ ; one with  $z > h_1$  has risk term at least  $nr_u(1 - h_1/n) \geq \Psi(h_1)$ . Consequently the analytic maximum occurs at  $k = n, m = G$  if

$$\begin{aligned} v - d &> q\alpha d, \\ 2(v - d) - c &> q\alpha(2d + F + R). \end{aligned}$$

For strictly concave  $u$  and  $h_F = nd - B_G > 0$ , every candidate defining  $\Psi(h_F)$  has positive cost: productive loss at  $z=0$ , tangent loss at  $z>0$ . Compactness makes the minimum positive. A funded offer with sufficiently small excess compensation therefore gives strict separation. The quantifiers are: for each fixed strictly concave utility, a suitable positive-margin offer exists.

Appendix B specifies that offer for four positions and two service groups. At its explicit endpoint,  $a = 19/20$ ,  $v = 1/2$ ,  $y_H = 34/25$ ,  $y_L = 117/100$ ,  $\Pr(H) = 3/4$ ,  $g = 1/10$ ,  $F = 3/10$ ,  $R = 1/5$ ,  $H = 1/20$  and  $K = 5001/10000$ . Independent post-decision opportunity and recovery Bernoulli halves give  $q = 1/64$  for every initially selected subset.

Each original creditor receives  $1 + 1/1000000$  statewise; each later financier receives more than its actual unit. Services earn expected fees 0.45005 against burden 0.45 and retain at least 33027/200000 terminal cash. Their acceptance uses expected wealth, not arbitrary risk-sensitive utility. Each owner's expected value is 0.5874615, exceeding the high/low waiting bounds 0.41 and 0.22. Appendix C proves subscription under binding pre-type procurement and visible sequential signatures, then best-route continuation; it does not solve unrestricted offer competition.

Actual date-0 inputs, setup and recovery reserves precede four separate date-1 inputs. Appendices D–F prove, separately, prefix conservation, beneficiary-level completed payoffs and attainment with

timely inputs, truthful certification, qualified recovery and prescribed scheduling. Their global funding and recovery-resolution guards prevent duplicated receipts and premature reserve release. Safe base-route choices need not attain the best-route value.

Appendix B's affine offer family has  $\pi_\tau = 77\tau/500000$ ,  $0 < \tau \leq 1$ , with positive owner waiting slacks and unconditional participant acceptance throughout. Choosing  $\tau < \min\{1, q\Psi(h_F)/(77/500000)\}$  establishes the claimed existence without changing resources or technologies; zero is only a limiting endpoint.

At  $\tau = 1$ ,  $W = 29/20$ ,  $\pi = 77/500000$  and  $U_A = 1174923/500000$ . For  $u(x) = -e^{-3x}$ ,  $e^s \geq 1 + s + s^2/2$  for  $s \geq 0$  gives

$$\Psi(h) \geq \min_{0 \leq z \leq h} \{\alpha(h - z) + 3z^2/(2n)\}.$$

At  $\alpha = 5/68$  and  $h = 999/5000 > n\alpha/3$ , multiplying this quadratic minimum by  $q$  gives  $38449/221952000$ . Hence

$$U_L^{\sup} \leq \frac{521548751}{221952000} < \frac{1174923}{500000},$$

The exact gap is  $533549/27744000000 > 0$ . The small margins prove a nonempty region; they do not calibrate commercial gains or implementation costs.

### 3.3 A pooled alternative and the boundary of the comparison

Appendix H constructs a fully funded pooled policy within L. It uses natural base receipts first. Only when every superior route is available does it divert  $(1/5)/\max_i y_i$  of the highest-output project's input to base production, supplying the original creditors' missing principal. The remainder funds pooled later repayments, contingent service fees and positive owner residuals. Reserves are paid once, and all participants accept under the same criteria; original creditors accept at equality.

Expected productive loss is  $\Gamma = \frac{1763}{1916928}$ . The policy attains  $U_{\text{pool}} = \frac{2814206413}{1198080000}$ . Section 3.2 gives  $\bar{U}_L = \frac{521548751}{221952000}$ , hence

$$U_{\text{pool}} \leq U_L^{\sup} \leq \bar{U}_L < U_A.$$

The advantage over the best policy in L lies between the positive certificate in Section 3.2 and  $27427067/29952000000$ . This brackets one coordinated construction against the comparison optimum, not two fully optimized institutional frontiers.

**Boundary control.** Give original creditors an enforceable  $1/20$  supplement from each later receipt, including a superior receipt's baseline component. Keep inputs, setup, recovery, later repayments and service fees unchanged. Each original creditor then receives one, and each owner receives  $1/1000000$  more than in the coordinated offer: total owner value  $2.349850$  rather than  $2.349846$ . This funded policy is outside L. Access to baseline receipts already provides the relevant capability; sharing the superior increment is unnecessary.

### 3.4 Protection, productive use and scale

For  $u(x) = -e^{-\gamma x}$ ,  $\gamma > 0$ , the tangent penalty is

$$R_\gamma(z) = nr_u(1 - z/n) = -z + \frac{n}{\gamma}(e^{\gamma z/n} - 1).$$

Differentiating  $\alpha(h - z) + R_\gamma(z)$  gives the strictly increasing  $e^{\gamma z/n} - 1 - \alpha$ . Thus

$$z^*(h) = \min\{h, \bar{z}\}, \quad \bar{z} = \frac{n}{\gamma} \log(1 + \alpha),$$

and

$$\Psi(h) = \begin{cases} -h + \frac{n}{\gamma}(e^{\gamma h/n} - 1), & 0 \leq h \leq \bar{z}, \\ \alpha h - \frac{n}{\gamma}[(1 + \alpha) \log(1 + \alpha) - \alpha], & \bar{z} < h \leq n. \end{cases}$$

If  $\bar{z} \geq n$ , only the first branch applies. Below the threshold, the minimizing decomposition uses creditor-risk loss; above it, marginal deficiency costs the productive-sacrifice slope  $\alpha$ . Greater risk aversion lowers the threshold. This describes the bound, not an attained allocation: its generous source inventory need not satisfy all other constraints jointly.

Substituting  $z = Gw$  gives  $\Psi_{u,2G}(Gh^*) = G\Psi_{u,2}(h^*)$  for nonnegative per-group deficiency  $h^*$ . A uniform positive per-group guarantee additionally needs the full-allocation slope conditions, a positive lower bound on  $q_G$ , compensation below that probability times the two-position penalty, and proportionally funded simultaneous recovery. Independent replication with a fixed compensation floor need not satisfy these conditions.

### 3.5 Comparison with related financial-contracting results

Financial relationships affecting asset value are established theory. Duffie, Gârleanu and Pedersen (2002, Proposition 1, equations 8–9) capitalize securities-lending income. Their Propositions 6–7 show non-monotone price effects and vanishing fees in the frictionless-search limit when lender bargaining power is below one. Gârleanu and Pedersen (2011, Proposition 5, p.1992) show weakly lower prices for persistently higher-margin identical-cash-flow assets when the constrained investor is long both; strictness requires a binding future margin constraint with positive probability.

Table 2: Prior conclusions and the specific question addressed here.

| Primary result                                             | Established conclusion or mechanism                                                                     | Present comparison                                                                                                               |
|------------------------------------------------------------|---------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------|
| Leitner (2010), Propositions 3/5 and Corollary 2, pp.12–18 | Reporting and collusion-resistant coordination affect collateral and investment under hidden contracts. | Certified opportunities and concave creditor participation yield a joint productive-use/risk penalty under a source restriction. |

| Primary result                                                      | Established conclusion or mechanism                                                                                           | Present comparison                                                                                                                                                          |
|---------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Bennardo–Pagano–Piccolo (2013 working paper), Section 6.2, p.28     | Timely exposure information and enforceable liquidation/repayment covenants can reproduce exclusivity.                        | Retain a same-input, higher-surplus later activity while bounding original-creditor repayment sources.                                                                      |
| Biais et al. (2006), Propositions 1–2                               | Dynamic agency contracts use reserves and payout/liquidation rules under hidden cash flow and unlimited risk-neutral finance. | Certified outputs, finite actual funding and paid recovery support a matched construction.                                                                                  |
| Duffie–Gârleanu–Pedersen (2002), Proposition 1 and Propositions 6–7 | Lending revenue enters price; bargaining and search complicate comparative price and fee effects.                             | Derive one funded authorization-to-execution change, apply the established equilibrium and separate shareholder capitalization, borrower benefits and real formation costs. |
| Gârleanu–Pedersen (2011), Propositions 2/4/5, pp.1990–1992          | Funding shadow costs and margins affect returns, bases and prices of identical-cash-flow assets.                              | Derive a source/protection/use bound; the source restriction is not an estimated margin or a price effect.                                                                  |

Two direct reductions fail. Affine creditor utility eliminates our tangent gap and strict penalty, so substituting Leitner’s risk-neutral preferences does not recover this certificate. Bennardo–Pagano–Piccolo’s larger project requires more input and has lower net surplus; a common positive change of units cannot make it our same-input, higher-surplus route. These reject particular mappings, not broader antecedents.

Similarly, lending capitalization alone supplies neither unconditional creditor participation nor the effective-source and dated-resource premises. Gârleanu–Pedersen’s margin is a portfolio constraint in equilibrium optimality conditions; our shortfall  $h$  enters a protection/use bound. The source shortfall and the equilibrium margin therefore play different analytical roles.

Propositions 3 and 4 use established valuation tools; their additional contribution is the funded admission or execution mechanism. Section 5.3 compares implementations. The DeMarzo–Fishman comparison is limited to the abstract record identified in the bibliography.

#### 4. Independent provision around continuing positions

The next results study a different margin. Once a position carries inherited claims, can another provider add a compatible service without reopening those claims or acquiring general control? The answer depends on the service’s incremental value, its actual setup and recovery resources, and the allocation of benefits required for participation. This is the paper’s entry and formation problem.

##### 4.1 Specialist entry after commitment

This is a separate continuation economy with a specialist’s actual capital  $K_e$ , idle in the no-entry benchmark. At admission, a finite remaining state space has public positive conditional probabilities and finite cash profiles. Inherited beneficiaries, state-contingent rights  $b_j(\omega)$ , priorities, remedies

and dates are binding. The best no-entry continuation in this domain supplies  $Y_0(\omega) \geq \sum_j b_j(\omega)$  and owner residual  $r_0$ .

The inherited contract preauthorizes an additive service with certified  $\Delta(\omega) \geq 0$ , unchanged inherited execution and junior fee  $0 \leq f(\omega) \leq \Delta(\omega)$ . Owner/entrant consent is required; an outstanding incumbent veto cannot be removed retrospectively. The entrant has no other revenues or credit, pays  $F_e \geq 0$ , locks separate  $R_e \geq 0$ , and pays recovery  $0 \leq c_e(\omega) \leq R_e$  before fees, from  $K_e \geq F_e + R_e$ . It requires expected final wealth  $K_e + H_e$ ,  $H_e \geq 0$ . Certification, binding work and timely performance are premises. The owner has strictly increasing statewise cash utility, with other benefits unchanged. Put  $D_e = \mathbb{E}[\Delta]$  and  $C_e = F_e + \mathbb{E}[c_e] + H_e$ .

**Proposition 2 (compatible post-commitment entry).** A fee preserving inherited rights and statewise owner protection, satisfying service acceptance and strictly improving the owner exists exactly when  $D_e > C_e$ . Requiring service margin  $\varepsilon > 0$  changes the condition to  $D_e > C_e + \varepsilon$ .

**Proof.** Entrant wealth is  $K_e - F_e - c_e + f$ , so  $\mathbb{E}[f] \geq C_e$ . The owner gains  $\Delta - f \geq 0$ ; strict preference requires a positive gain in a positive-probability state. Thus  $D_e > \mathbb{E}[f] \geq C_e$ . Conversely, choose  $q_e \in [C_e, D_e)$  and  $f = q_e \Delta / D_e$ . This satisfies every state's cap, service acceptance and strict owner improvement;  $q_e > C_e$  gives positive service profit. Earlier work and recovery use actual  $K_e$ , not the promised fee. Replace  $C_e$  by  $C_e + \varepsilon$  for the margin case. QED.

Appendix G realizes the result with product state (B,Z): inherited commands retain their rules, extension-only commands affect a separately funded Z, and deleting the latter projects to a valid inherited execution. Junior claims survive handoff/closure, while old closure never waits for the new service. Timely completion and physical capacity remain separate from logical preservation. This additive domain does not establish arbitrary collateral substitution, liquidation amendments or physical option delivery.

## 4.2 Timely service and voluntary formation

Advance acceptance can recover an expiring opportunity. In Appendix J's example the owner has free cash  $W = 7/2$ , the specialist has four units, and urgent/ordinary opportunities are equally likely. Urgent work costs four and returns sixteen; ordinary work costs two and returns six. The same funding and work take one unit of time. Fresh consent adds one unavoidable unit before work can begin. Deadlines are one and three, respectively; an accepted charter costs one initially. Lemma G.1 proves this dependency-based completion difference with unchanged inherited claims and actual funding.

Table 3: Recovered service surplus and its distribution.

| Quantity                        | Fresh consent                            | Accepted charter                     |
|---------------------------------|------------------------------------------|--------------------------------------|
| Urgent service                  | Expires; specialist capital remains idle | Work four paid before output sixteen |
| Ordinary service                | Work two paid before output six          | The same work and output             |
| Expected owner service cash     | $2(1 - \beta)$                           | Eight                                |
| Expected incumbent consent cash | $2\beta$                                 | Zero                                 |
| Initial real setup              | Zero                                     | One                                  |
| Specialist terminal cash        | Four in each state                       | Four in each state                   |

The provider receives its work cost after output, restoring its four units. If the incumbent's ordinary consent fee is fraction  $\beta \in [0, 1]$  of surplus four, expected owner and incumbent gains after setup and upfront compensation  $x$  are  $5 + 2\beta - x$  and  $x - 2\beta$ . Their sum is five: six of new urgent surplus less setup one. For every  $\beta$ ,  $x = 2\beta + 1/2$  fits the same  $W$  and gives gains  $9/2$  and  $1/2$ . Initial charter acceptance is ex ante and risk-neutral, unlike Proposition 2's later statewise protection.

More generally, with urgent probability  $p$ , net service surpluses  $s_u, s_o$  and setup  $M$ , voluntary formation in this nonnegative upfront-transfer class is possible exactly when  $ps_u > M$  and  $W \geq M + (1 - p)\beta s_o$  (Corollary J.2). Productive value and formation liquidity are separate conditions. For another fully funded arrangement completing both services with identical claims and provider terms at date-0 cost  $C_*$ , the charter's joint advantage is  $C_* - M$ . A timely consent alternative can therefore eliminate the advantage. This deadline example is not Theorem 1's policy-wide comparison or a calibration of institutional latency.

The formation comparison distinguishes protection of an accepted financial claim from control over admission of an additional service. In the specified domain, inherited claims remain protected while advance acceptance removes the need for fresh consent. The incumbent may nevertheless lose consent income. That loss affects compensation and the liquidity required for voluntary adoption; it is distinct from the productive surplus recovered by completing an otherwise expiring opportunity. The result identifies a distributional obstacle to changing authority arrangements without treating incumbent consent rights as evidence of misconduct.

### 4.3 Transferable service value

**Proposition 3 (formation and capitalizable entitlement).** Use Appendix J.2's funded urgent/ordinary technology, with common positive valuation weights  $\lambda_u + \lambda_o = 1$  in a zero-interest numeraire; these need not equal physical probabilities. Let  $s_u, s_o$  be owner residuals after all provider payments. Fresh consent completes ordinary work and pays incumbent fee  $\beta s_o$ ; the charter completes both with the same inherited claims and remedies. Set  $r_\lambda = \lambda_o \beta s_o$  and  $G_\lambda = \lambda_u s_u + r_\lambda$ . With actual owner cash  $W$ , setup  $M$  and nonnegative upfront compensation  $x$ , strict owner gain and weak incumbent acceptance are feasible exactly when  $\lambda_u s_u > M$  and  $W \geq M + r_\lambda$ . Their joint valuation gain is  $\lambda_u s_u - M$ ; provider margins enter separately when aggregating all participants.

If the installed entitlement and burdens pass to an eligible buyer with unchanged execution, funding and service terms, its cash-flow valuation premium is  $G_\lambda$ . The originator retains  $G_\lambda - M - x$ . A personal entitlement implies no premium for the bare security. Transaction-price interpretation additionally requires that this functional price the relevant tradable claims consistently with feasible replication. Appendix J.3 supplies the proof, cash-flow vectors and conversion bound.

At Section 4.2's primitives,  $\lambda_u = \lambda_o = 1/2$ ,  $\beta = 1/2$  and  $x = 3/2$  give installed premium seven: six of new urgent surplus plus one reassigned fee. Setup and compensation leave the owner  $9/2$  and incumbent  $1/2$ , with joint gain five. The buyer must not deduct already paid formation costs again; a seller cannot count both sale proceeds and the receipts sold. At  $\lambda_u = 1/20$ , urgent surplus instead has value  $3/5$ , below setup one, although physical expected surplus is still five. Capitalization depends on where value occurs and who receives it.

## 5. Stock-market incidence and lifecycle implementation

Stocks make the organizational question concrete because financing, lending, execution and delivery can attach to the same position while issuer cash flows remain unchanged. The economic application

uses a financed share whose lender accepts a funded substitution rule in advance. It traces the chain from authority to an executable opportunity, from execution to market quantity, and from bargaining to capitalization and borrower rents. The physical-return illustration then tests whether a later use can complete without erasing the inherited share-return claim. These are separate constructions: neither local implementation establishes the market equilibrium or measures its price effects.

### 5.1 Executable stock lending and endogenous valuation

Proposition 3 fixes valuation weights. Appendix L instead solves a stock-lending market: an accepted substitution rule permits a bank-financed share to be lent after the borrower funds a replacement payment source preserving the bank's claim. Fresh approval can miss an expiring contact. The construction changes executable contacts while holding raw counterparties and issuer cash flows fixed, and applies Duffie, Gârleanu and Pedersen's (2002) established lending equilibrium.

**Proposition 4 (funded execution and stock-price incidence).** In Appendix L's separate economy,  $n_s > 0$  is physical float,  $n_b > 0$  the finite eligible short-seller mass,  $\gamma > 0$  redemption intensity and  $\eta \in (0, 1)$  lenders' bargaining share. Optimists value issuer payments at  $V_H$  and have sufficient mass for  $n_s + n_b$  long exposures; pessimists value them at  $V_L$ . Each loan consumes service expenditure  $c \geq 0$ , with  $D = V_H - V_L - c > 0$ . Uniform transferable access and independent authorization success give  $\lambda = a\bar{\lambda}$  from raw intensity  $\bar{\lambda}$ . Initial price and expected completed loans are

$$P_0(\lambda) = V_H + \ell_0(\lambda), \quad \ell_0(\lambda) = \frac{\eta n_b \gamma D \lambda}{(\gamma + n_s \lambda)(\gamma + (1 - \eta)n_s \lambda)}, \quad Q(\lambda) = \frac{n_b n_s \lambda}{\gamma + n_s \lambda}.$$

For  $\lambda_1 > \lambda_0$ , quantity increases; price increases exactly when  $\gamma^2 > (1 - \eta)n_s^2 \lambda_0 \lambda_1$ , is unchanged at equality, and otherwise falls. Initial owners' net adoption value is  $n_s \Delta P - K$  for their actual funded setup K. Bank claims remain unchanged and providers recover  $c$ . Appendix L derives borrower benefits and proves market clearing and participation under the declared restrictions; the comparison is between uniform regimes, not a decentralized adoption equilibrium.

With  $n_s = n_b = \gamma = 1$ ,  $\eta = 1/2$ ,  $V_H = 110$ ,  $V_L = 98$ ,  $c = 2$ , raising  $\lambda$  from  $1/10$  to  $1/2$  gives price gain  $208/231$  and owner gain  $809/1155$  after  $K = 1/5$ . Raising it from  $2$  to  $4$  instead lowers price by  $1/3$  while lending increases. Corollary L.4 admits a positive cost of net locked liquidity; its positive example retains owner gain  $601/1155$  after setup.

The economic domain requires funded cash-closeout loans, fixed eligible populations, uniform access, zero safe interest, no freely issued derivative substitute and no omitted incumbent consent rent. Each borrower posts a distinct actual reserve. These are analytic outcomes, not measured implementation-specific premiums. Theorem 1's fixed-resource value gain, minimum capital for an identical protected outcome and market-wide competition remain different claims. Section 5.2 addresses physical delivery rather than this cash-closeout market.

### 5.2 Physical delivery while preserving share return

A stockholder lends a homogeneous share position; the borrower then sells a call with delivery-or-strike-refund terms. The stockholder retains a dated equivalent-share return claim. Physical option delivery requires purchased, reserved replacement shares; otherwise an unfilled funded exercise

receives its strike back after the window, without premium refund or compensation for missed delivery. One share cannot discharge two incompatible deliveries.

In the local implementation, the original position reaches the option buyer only as replacement shares reach the stockholder atomically. The controller prevents reserved replacements from being relended. Call expiry plus the full delivery window must precede or equal the inherited loan due date, and an unaccepted offer must be terminable at expiry. Appendix K proves the resulting timing and two-call return remedy under the stated dependencies.

The illustrative borrower pays 12,000 for 100 replacement shares before receiving the escrowed 11,000 strike. Those replacements already exist in shop inventory. After loan fee and premium, the exercised path loses 900; mechanical compatibility is not proof of ex-ante profitability.

The local tests demonstrate delivery coordination, not unified order-book liquidity or live stock-market effects. Appendix K gives the full cash ledger, rejected failure cases and asset domain.

### 5.3 Implementation conditions and antecedents

The mechanism does not depend on one technical substrate. Assignments, covenants, common registers, delegated accounts or commitment systems can implement it when they separate service authority from amendment of accepted claims, mediate every relevant asset path and preserve funded remedies. ERC-7710 and ERC-6900 provide delegation, validation and management interfaces; they do not by themselves specify financial beneficiaries, dated resources or claim-preserving administration. A conforming implementation can. A bypass that drains backing while leaving an entitlement record cannot.

External recognition is therefore part of the economic mechanism rather than a display property of the ledger. A creditor, issuer, custodian, insurer or successor service must treat the continuing state as relevant to its own decision. Technical evidence can reduce verification cost, but it cannot by itself create legal priority, qualification, loss-bearing capacity or an obligation that the governing arrangement does not assign. Institutionally portable relationships require both persistent state and an accepted recognition path.

The native-share prototype used here separates a position register, a fixed authority core and service applications. Bound positions consult the core on owner transfers; applications request specified actions without direct powers over claim state or unrestricted transfers. Within its fixed loan/call grammar, a lender can preaccept qualifying calls, another application can admit one without a new lender message, funded exercise pays loan priority, and an accepted default sale preserves the live call. Revoking unused authority leaves accepted claims intact. This construction differs from Section 5.2’s equivalent-share return arrangement and from the private-state construction used for write-domain separation.

There are substantive antecedents and alternatives. Daml provides accepted obligations, persistent bounded authority, holdings, locks, loans, physical options and atomic settlement. EVC separates revocable operators from creditor controllers, composing interacting claims within a final account controller. Zexe constrains private-record consumption and permits outsider execution of conditional orders; its published proof delegation and exchange use different authorization and cancellation rules from the encumbrance construction studied here (Bowe et al., 2020, Sections 4–6 and Appendix B.2; Hauser, 2026, Sections 3–4). These differences identify implementations, not exclusive economic capabilities. Integrated institutions may realize equivalent rights. Write-domain separation alone implies neither the repayment-source restriction nor a value gain; the economic

results require the additional domains and conditions stated in this paper.

## 6. From asset transferability to relationship portability

Three changes should be distinguished. Digitization changed how financial records are represented and processed. Tokenization can make asset state natively transferable and programmable. Shared authority addresses a further organizational problem: whether the claims, mandates, priorities and remedies around that state can remain effective as uses or operating providers change. The third change does not follow automatically from the first two. It requires accepted commitments, complete mediation, funded performance and recognition by the actors whose decisions give the position financial effect.

In provider-centered organization, the account provider commonly combines service provision with control over the authoritative record, qualification and continuity of the relationship. Changing providers can therefore require the position and its inherited terms to be closed, transferred or reconstructed. Position-centered organization does not eliminate the provider. It changes the provider's possible role. A lender, custodian, insurer, broker or service operator may still supply capital, judgment, risk bearing, loss absorption and legal responsibility. Where accepted rules and recognition permit succession, however, the provider can act as a qualified officeholder exercising a bounded mandate over a continuing position rather than as the exclusive keeper of the relationship's institutional memory. Replacing that officeholder does not automatically transfer its balance-sheet liabilities or other financial obligations.

Recognition determines whether this organizational possibility becomes portable in practice. A technically open execution layer is not economically contestable if one actor exclusively determines which positions count, which evidence is accepted, who qualifies to serve them, which amendments bind and who may succeed an incumbent. The recognition layer can therefore become a new institutional chokepoint even when service execution is distributed. Conversely, recognized common state and governed succession can reduce the need to reconstruct a relationship inside every provider's private perimeter. The relevant policy and market-design questions concern who sets these rules, how decisions can be challenged and whether qualification and succession remain contestable.

This discussion yields a transition hypothesis rather than a universal prediction. If position-centered organization becomes economically important, qualified outside providers should begin relying on continuing positions and their inherited claims without requiring complete migration, re-underwriting and reconstitution. Its emergence should be selective: integrated provision should remain where joint netting, balance sheet, liquidity, judgment or legal responsibility creates value. The empirical object is therefore not the disappearance of intermediaries but a change in which functions require control of the account and which can be supplied through bounded authority around a recognized position.

## 7. Conclusion

This paper develops shared authority as an economic mechanism for organizing compatible financial relationships around a continuing position. Its design space has two dimensions: services that can coexist and commitments that survive specified changes in use or operator. Within declared domains, the financing theorem quantifies a protection/use trade-off and constructs a funded arrangement exceeding the resulting policy-wide bound; the entry and formation results characterize compatible specialist provision around inherited claims; and the valuation results connect executable opportunities to activity, rents and capitalization.

Tokenization can make an asset transferable; it does not by itself make the financial relationships around that asset portable. Institutional portability additionally requires recognized state, assigned responsibility, funded performance and remedies. Failure supplies the demanding test: after an operator is removed, the position should remain defined, valid claims and priorities should survive, remedies should remain reachable and a qualified successor should be able to act under accepted terms. The constructions in this paper establish bounded parts of that test, not general financial-counterparty succession or universal portability across legal and custody systems.

The resulting market prediction is falsifiable. If shared authority matters economically, independent qualified providers will begin changing financing, protection, execution or servicing decisions in reliance on continuing positions without first rebuilding those positions inside provider-controlled accounts. If every new service still requires migration and complete reconstruction of the relationship, tokenization has transferred the asset without making its financial relationships portable.

## Appendix A. Assumptions and proof map

Sections 2–3 state all premises and prove the policy-wide bound. The following appendices verify its coordinated offer and distinguish safety, completed payoffs and attainment. The comparator is not restricted to that offer’s contract, scheduler or interface.

Table 4: Proof and implementation dependencies.

| Main-paper claim                                                                         | Supporting argument here | Additional conditions                                                                                                       |
|------------------------------------------------------------------------------------------|--------------------------|-----------------------------------------------------------------------------------------------------------------------------|
| A positive-margin offer exists for each fixed strictly concave original-creditor utility | B–C                      | The stated financial primitives, original-creditor preferences and expected-wealth service acceptance                       |
| The offer is subscribed and continued                                                    | C                        | Binding pre-type procurement, visible sequential signatures, later certification and the declared continuation alternatives |
| Every admitted prefix preserves resources and accepted rights                            | D                        | Authoritative atomic interface, actual unique inputs, truthful certificates and one-failure recovery domain                 |
| Completed executions have the constructed beneficiary payoffs                            | E                        | Every required claim, recovery and closure obligation is completed                                                          |
| The best-route policy attains the economic value                                         | F                        | Timely inputs, certification, available roles and recovery, phase-compatible scheduling and prescribed route choices        |
| Additive entry preserves inherited execution and attains the new payoff                  | G                        | Preauthorized product class, disjoint new capital, unchanged old rules and timely new work                                  |

The recovery law and expected-wealth service criterion apply to both economic arrangements. Binding pre-type procurement restricts the constructed offer, not every comparison policy. Actual unique arrivals, truthful certification and qualified paid recovery govern execution; timely inputs and prescribed operations are additional attainment premises. Proposition 2 uses separately funded capital and an already accepted additive charter.

## Appendix B. The funded offer family and its endpoints

For four positions and two groups, fix

$$\begin{aligned} a &= 19/20, \quad v = 1/2, \quad y_H = 34/25, \quad y_L = 117/100, \\ \Pr(H) &= 3/4, \quad g = 1/10, \quad F = 3/10, \quad R = 1/5, \\ \delta &= 1/2, \quad H = 1/20, \quad K = 5001/10000. \end{aligned}$$

H in a type label denotes high output; standalone H is the service burden. Type draws are independent; opportunities and recovery follow the declared law. For  $0 < \tau \leq 1$ , set

$$\begin{aligned} s_\tau &= 1/20 + \tau/1000000, & p_{H,\tau} &= 1 + \tau/100000, \\ p_{L,\tau} &= 1 + \tau/50000, & e_\tau &= 3/80 - 77\tau/2000000, \\ f_{H,\tau} &= 109/400 + 11\tau/400000, & f_{L,\tau} &= 33/400 + 7\tau/400000. \end{aligned}$$

Substitution gives  $y_t = p_{t,\tau} + s_\tau + f_{t,\tau} + e_\tau$ ; superior production adds  $g$  to the owner's residual from the same input. Each affine quantity equals  $(1 - \tau)A(0) + \tau A(1)$ , so the endpoints establish positivity throughout:

Table 5: Affine offer endpoints.

| Quantity                          | $\tau=0$ , limiting value | $\tau=1$          |
|-----------------------------------|---------------------------|-------------------|
| Base owner residual $e$           | $3/80$                    | $74923/2000000$   |
| High-type fee                     | $109/400$                 | $109011/400000$   |
| Low-type fee                      | $33/400$                  | $33007/400000$    |
| Expected owner value $v+e+g/2$    | $47/80$                   | $1174923/2000000$ |
| Owner gain over high-type waiting | $71/400$                  | $354923/2000000$  |
| Owner gain over low-type waiting  | $147/400$                 | $734923/2000000$  |
| Lowest terminal service cash      | $1651/10000$              | $33027/200000$    |

Two low-type fees and recovery minimize service cash:

$$K - F - R + 2f_{L,\tau} = 1651/10000 + 7\tau/200000.$$

Before fees,  $K$  funds setup and the distinct reserve, leaving buffer  $K - F - R = 1/10000$ . This proves nonnegative service wealth, not statewise principal protection. Each original creditor receives  $a + s_\tau = 1 + \tau/1000000$  statewise; each later financier receives more than one. Expected two-position service fees are

$$2[(3/4)f_{H,\tau} + (1/4)f_{L,\tau}] = 9/20 + \tau/20000.$$

Since  $F + \delta R + H = 9/20$ , the service exceeds its expected-wealth reservation by  $\tau/20000$ . Arbitrary risk-sensitive service utility need not accept. Total excess compensation, deducted from owner value, is

$$\begin{aligned}
\pi_N &= 4\tau/1000000, \\
\pi_T &= 4\tau[(3/4)/100000 + (1/4)/50000], \\
\pi_S &= 2\tau/20000, \\
\pi_\tau &= 77\tau/500000.
\end{aligned}$$

H is a participation burden, not another settlement payment. The zero endpoint is only a limit. For each fixed strictly concave increasing creditor utility,  $q\Psi(h_F) > 0$ ,  $h_F = 999/5000$ . Choosing

$$0 < \tau < \min\{1, q\Psi(h_F)/(77/500000)\}.$$

makes compensation smaller than the comparison penalty. The same primitives satisfy Section 3.2's slope conditions. This proves utility-by-utility existence, not a uniform positive margin across all preferences.

### Appendix C. Procurement, acceptance and continuation

Providers bind the conditional package before private types and close the offer stage. Each owner then observes its type; later truthful public certification determines the fee branch. Unverified reports cannot alter payouts. A two-position group installs only after sequential visible signatures; noninstallation preserves later-only opportunities.

Either type obtains  $v + e_\tau + g/2$ , exceeding waiting bounds  $y_H - 1 + g/2 = 41/100$  and  $y_L - 1 + g/2 = 11/50$  by Appendix B's positive endpoint slacks. The last owner signs; anticipating this, the first signs. Subscription therefore leaves the type distribution unchanged, so the service evaluates its pre-type commitment under the original prior. Appendix B verifies all other participants' acceptance at their decision dates.

After installation, initial input/setup are sunk. Base and superior owner residuals are respectively  $e_\tau > 0$  and  $e_\tau + g$ ; later financiers receive  $p_{t,\tau} > 1$  from the declared source. The enforceable priorities and continuation alternatives support funding and the owner's best-route choice. Successor availability remains a separate premise. This procurement argument does not derive unrestricted competing offer prices or rule out asymmetric rival arrangements.

### Appendix D. Prefix preservation by induction

Each of G two-position roots records actual initial N/S cash, fixed beneficiaries/terms, certificates, later deposits, routes, output escrows, unpaid claims, operator epochs, one-use reserves, recovery resolution and closure. A global book records genuine dated arrivals, unused financier cash and unique source-to-position associations. Date-1 promises are absent from date-0 balances.

The following are the *only* asset-affecting transitions: withdrawal, upgrade and emergency paths cannot bypass them. Invalid requests leave state unchanged.

Table 6: Guarded execution transitions.

| Transition                    | Guard and effect relevant to the invariant                                                                                                                                                                  |
|-------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Registration and installation | Register only declared resources and distinct identities; obtain required consent, pay real setup and initial inputs, lock R and freeze accepted terms.                                                     |
| Observation and phase change  | Reveal truthful certificates without creating cash. Leave date 0 only after all declared installations; leave date 1 only after observation, actual funding and admission, with recorded failures resolved. |
| Actual arrival                | Credit a genuine permitted dated unit once to the global book.                                                                                                                                              |
| Later funding                 | Atomically debit an unspent arrived unit and credit one position; fix its unique association. Failure leaves both accounts unchanged.                                                                       |
| Route admission               | Require certification, availability, consent, the funded input and the full claim-source inequality; spend the input and admit the route once.                                                              |
| Maturity                      | Credit only declared output of funded projects; create the accepted matching claim partitions once.                                                                                                         |
| Settlement                    | Debit the appropriate escrow and pay one stable unpaid claim identity once.                                                                                                                                 |
| Failure and handoff           | Fence a failed epoch; pay the one permitted qualified recovery from its locked reserve; change the operator epoch without changing beneficiaries or financial terms.                                        |
| Resolution and closure        | Resolve the recovery window truthfully; close only after all claims settle and any required recovery is paid. Return only an unused reserve.                                                                |

The invariant asserts nonnegative balances; unique input consumption; conservation of initial cash, genuine arrivals and funded output net of real expenditure; immutable accepted claims; source-bounded unpaid obligations; exactly-once settlement; and one-use reserves.

Registration establishes it. Installation spends available funds and partitions the reserve. Information/phase steps create no cash. For cumulative arrivals  $E$ , book cash  $B$  and transfers  $F_r$ ,  $B + \sum_r F_r = E$ : arrivals raise  $B$  and  $E$  equally, and funding lowers  $B$  while raising one  $F_r$ . Spending inside a root does not erase its historical association, so the same receipt cannot fund two roots.

Admission spends one deposited input. Maturity creates only its declared output and matching bounded claim partitions. Settlement reduces escrow and unpaid obligation equally. Handoff spends the reserve once while preserving financial identities. Closure requires the irrevocably resolved recovery outcome and any required payment; it cannot cancel recovery by preceding resolution. Rejections stutter. These exhaustive cases prove the invariant by induction for every finite prefix and interleaving.

The proof requires the global funding and recovery-resolution guards; earlier executors lacking them are counterexamples, not evidence for this stronger claim. The 1/10000 service buffer is an explicit initial resource, retained once and never pledged to another use.

## Appendix E. Completed-payoff correspondence

Let  $D_r \in \{0, 1\}$  indicate required recovery and  $\rho_i \leq \eta_i$  the chosen superior route and its availability. These local indicators differ from Section 3.1's allocation fractions. Closure and immutable terms give

$$\begin{aligned} X_{N_i} &= a + s_\tau, \\ X_{T_i} &= p_{t_i, \tau}, \\ X_{S_r} &= K - F - RD_r + \sum_{i \in r} f_{t_i, \tau}, \\ C_i &= y_{t_i} + g\rho_i - p_{t_i, \tau} - s_\tau - f_{t_i, \tau}. \end{aligned}$$

Owner value adds  $v$  to cash  $C_i$ . With  $J = R \sum_r D_r$  and  $Y = \sum_i (y_{t_i} + g\rho_i)$ , internal repayments and fees cancel:

$$\sum_i (C_i + X_{N_i} + X_{T_i}) + \sum_r X_{S_r} = GK - GF - J + na + Y.$$

This is the full-investment budget. Real input/setup/recovery recipients lie outside the financing network; a book transfer creates no extra endowment. The analytic correspondence holds beneficiary by beneficiary, independently of settlement order and qualified handoff.

A pooled executor must also bind root-local accounts to actual financiers through immutable beneficial ownership. Historical Python histories verified local accounts and totals but omitted that map; the separate local identity refinement tests the missing association. The analytic statement requires it regardless of those tests.

Safe completed base-route traces may yield less than the construction. Appendix C selects  $\rho_i = \eta_i$ , attaining  $n(v + e_\tau + g/2)$  under the opportunity law, or 1174923/500000 at  $n = 4, \tau = 1$ .

## Appendix F. Completion and its separate premises

Fix a permitted world. Assume genuine inputs/observations arrive in their phases, certification and recovery resolution are truthful, qualified actors/successors are available, and prescribed enabled work is eventually submitted. Phase barriers preserve unfinished funding but cannot supply absent external funds.

After registration, each root has 21 tasks: installation, observation, two fundings, two admissions, maturity, twelve claim settlements (including zero bookkeeping), recovery-window resolution and closure. Add 2G genuine arrivals, two phase advances, and two tasks per required failure/handoff. The finite remaining-task rank is

$$23G + 2 + 2 \sum_r D_r.$$

This fixed-world rank need not be known to actors. Every successful prescribed action removes a task; maturity claims were pre-counted, and rejections stutter. Phase/recovery dependencies expose the next task under the availability premises. Each root has at most one failure, exposing its funded handoff. Eventual execution exhausts the rank and, with Appendix E, attains the policy's payoff.

This proves neither completion of arbitrary safe traces nor wall-clock deadlines from weak fairness. Physical deadlines additionally need bounded arrivals, detection, recovery and operations with sufficient capacity and slack. Safety, completed-payoff correspondence and attainment are separate claims.

## Appendix G. Additive specialist entry: conservative extension

Proposition 2 assumes an accepted additive charter or the actual consent required to amend it. Write state  $(B, Z)$ , with complete inherited execution  $B$  and separate entrant account  $Z$ .  $Z$  records genuine  $K_e$ , paid  $F_e$ , reserve  $R_e$ , recovery  $c_e$ , incremental output  $\Delta$ , junior fee  $f$  and owner gain  $\Delta - f$ . Capital is idle in the no-entry benchmark. No extension command may spend inherited backing or amend its terms.

Inherited commands keep their exact acceptance rules and  $B$  updates. Maturity may additionally credit actual output to  $Z$  only after funded, admitted, timely new work. Extension commands affect only  $Z$  and require accepted authority, current history/epoch, immutable inherited terms, owner/entrant consent, actual  $K_e \geq F_e + R_e$  and  $0 \leq f \leq \Delta$ . Admission/output/payment flags prevent duplication. Junior payment follows relevant inherited settlement, but old settlement and closure never wait for  $Z$ . Accepted junior claims, payment authority and a qualified settlement role survive inherited handoff and closure; stale-epoch rejection cannot cancel them.

Under projection  $P(B, Z) = B$ , inherited steps remain old steps, extension steps stutter and rejections change neither component. In  $Z$ , admission pays work and locks a distinct reserve; recovery spends at most that reserve; funded output is credited once; settlement debits its escrow once; unused reserves are released only after irreversible resolution and payment of every required new recovery. Induction gives unchanged inherited rights/effects and distinct nonnegative new balances for every finite trace.

If inherited continuation and timely new work complete, with qualified junior settlement,

$$\begin{aligned}\text{Owner incremental cash} &= \Delta - f, \\ \text{Entrant final cash} &= K_e - F_e - c_e + f.\end{aligned}$$

These are Proposition 2's payoffs under identical resources and expected-wealth acceptance; services may lose wealth in some realized states. Late new work can forfeit its incremental payoff without blocking old settlement. Physical scheduling capacity is needed to preserve real deadlines. This is an analytic construction; an executable implementation of  $Z$  is not established here. Share transfers, collateral substitution and liquidation amendments alter  $B$  and require different compatibility proofs.

### G.1. Admission without a new incumbent response

At time zero an unforeseen service and verifiable terms become available. Two previously accepted charters have the same inherited  $B$ , backing and entrant capital. Fresh consent requires authenticated opportunity-specific approval before any payoff-producing work; no stored approval, substitute, alternative path or speculative pre-work is available. Earliest approval is  $\sigma > 0$ . The advance charter already accepts  $G$ 's additive class and can be invoked by consenting eligible owner/entrant without a further incumbent response; its formation remains subject to Appendix J.

Both paths require the same achievable minimum duration  $\tau > 0$  for validation, funding and full work after authorization. Capital, certificates, owner/entrant consent, qualified actors and capacity are available at zero; old deadlines/remedies remain achievable. Output exists only upon completion by  $d$ , with no partial-output alternative.

**Lemma G.1.** For  $\tau \leq d < \sigma + \tau$ , every fresh-consent execution misses the deadline while the advance charter has a funded compatible completion. If approval arrives at  $\sigma$  and  $d \geq \sigma + \tau$ , both can complete; if  $d < \tau$ , neither can.

**Proof.** Fresh-consent ordering forces earliest start  $\sigma$  and completion  $\sigma + \tau$ ; waiting, batching or operator substitution cannot bypass its guard. Advance class membership, consent and funding enable work at zero, completed at  $\tau$ . Projection preserves inherited financial effects; the capacity premise preserves dates. The other cases follow from the same duration bounds. QED.

At  $\sigma = \tau = 1$ , deadlines one and three give Appendix J's example when ordinary approval arrives at one. The gain removes a response dependency, not required work. A sufficient prior delegation or other accepted funded path can supply it; faster consent changes the comparison. This lemma covers its specified interface, unlike Theorem 1's financing-policy bound.

## Appendix H. A fully funded pooled policy

Use the four-position primitives of Appendix B at  $\tau=1$ . All type draws, superior-route availability draws and service-failure draws are mutually independent, with the stated probabilities. All four initial inputs, both service setups and both separate recovery reserves are funded. The pooled policy protects each original creditor at its outside wealth one, using only first and base receipts. It uses coordination among permitted sources, so it is not a benchmark without any coordination.

Original creditors require a later supplement  $D = 4(1 - a) = 1/5$ . After certification, if some superior route is unavailable, select the highest-output such position (breaking ties by index) and earmark  $D$  from its base output. Every position whose superior route is unavailable uses its base route, and all available superior routes are used. If all superior routes are available, select the highest-output position  $j^*$ , allocate the fraction  $D/y_{j^*}$  of its actual later input to base production, and allocate its remaining input and every other input to superior production. This creates  $D$  of base receipts. Each original creditor receives  $D/4$  plus  $a$ , hence one in every state. No superior receipt or recovery reserve pays that supplement.

Later finance receives the same type-dependent payments as Appendix B, from the pooled portfolio rather than each isolated project. Define the remaining actual cash

$$L(\omega) = \sum_i \text{later output}_i - D - \sum_i p_{t_i}.$$

Its statewise minimum is  $4(117/100) - 1/5 - 4(50001/50000) = 47992/100000 > 0$ . The two services split the fee pool  $\rho L$  equally, where

$$\rho = \frac{9001/10000}{\mathbb{E}[L]}.$$

The remaining  $(1 - \rho)L$  is divided equally among the owners. These contingent terms are binding before private types; certification determines payouts, not an owner's unverified report. The expectation calculation below gives  $0 < \rho < 1$ . Each original creditor weakly accepts statewise; each later

financier receives more than its actual input. Service  $r$  ends with  $K - F - RD_r + \rho L/2 \geq 0$  and expected fees 9001/20000, exceeding its expected burden 9/20 by 1/20000. Expected-wealth service acceptance is the same as in the coordinated construction. Each owner receives more than  $v=1/2$  in every financed state, exceeding the high-type waiting bound 41/100. Visible sequential signatures in a binding all-or-nothing pooled mandate therefore yield full subscription by backward induction. Indifferent original creditors accept under the declared weak-acceptance convention. The service commitment is conditional on subscription, and setup and reserve funding precede initial advances.

All later inputs are genuinely supplied. The accepted routing mandate enforces the base supplement and minimizes the associated diversion among its statewise-protected, fully funded continuations. It is not claimed optimal over every defaultable comparison policy. Each required recovery spends its own  $R$  once, preserves beneficiaries and contingent prices, and uses the same truthful certification and qualified-availability premises as the constructed coordinated policy.

Extra productive diversion occurs only when all four superior routes are available, with probability 1/16. Conditional on that event, the highest output is the high type unless every type is low, with probability 1/256. Thus

$$\Gamma = \frac{1}{16} gD \left[ \frac{255/256}{y_H} + \frac{1/256}{y_L} \right] = \frac{1763}{1916928}.$$

Undiverted expected later output is 109/20 and expected later repayment is 80001/20000. Consequently

$$\begin{aligned} \mathbb{E}[L] &= 24999/20000 - \Gamma > 9001/10000, \\ U_{\text{pool}} &= 2 + \mathbb{E}[L] - 9001/10000 = \frac{2814206413}{1198080000}. \end{aligned}$$

On the critical event, both recoveries are paid and actual base output  $D$  supports original creditors. The accessible-source inventory is  $4 - 4(1 - a) + 2K - 2(F + R) + D = 4 + 1/5000$ , enough for their four-unit payment, with no superior source. The all-policy upper bound therefore applies to this policy. Combining it with the simpler certificate in Section 3.2 gives the independently sufficient interval

$$\frac{533549}{27744000000} \leq U_A - U_L^{\text{sup}} \leq \frac{27427067}{29952000000}.$$

The upper endpoint equals  $\Gamma - 1/250000$ : forgone opportunity minus the four saved original-creditor margins. These are normalized theoretical values, not estimates of market gains. The interval concerns one coordinated construction and the best qualifying comparison policy, not two solved institutional frontiers.

## Appendix I. A restricted realization of the repayment boundary

This appendix supplies a sufficient contracting domain for the source premise. It does not derive that premise from separate ownership or programmatic assets. Let  $A$  contain all initial and unused financial balances, initial-project receipts and base-route receipts;  $B$  contains superior-route receipts. Original creditors' ultimate beneficiaries are the fixed set  $N$ . Impose the following explicit institutional primitives.

1. **Fixed, nonassignable B payees.** B's recognized payout names ultimate beneficiaries other than N. Credential transfers and wrappers cannot change that beneficiary or create an assignment, pledge, novation or enforceable nominee claim for N. This is a restriction of the modeled contract language, not of ordinary securities generally.
2. **No outside recourse to B.** Recognized contracts may bind A, including pooled balances and state-contingent guarantees, but cannot bind B or its recipient's subsequent wealth. No external judgment, lien, cross-default, setoff, seizure or terminal penalty reaches B. Any such capability must be included if present in an application.
3. **Terminal noncommitment.** B arrives after all inputs, required recovery, A settlements and other payoff-relevant exchanges are irrevocably resolved. Recipients strictly prefer more of their own retained cash, have no beneficial overlap with N, and have no remaining reciprocal service, reputation, altruistic objective or sanction supporting a discretionary onward payment. Contracting and payment histories are finite.
4. **Closed dated resources.** All earlier payments use declared A balances. There is no new endowment, supplier credit, uncouted collateral or outside financier. Intermediate consumption and reinvestment enter the same complete ledger as the theorem.

**Lemma I.1.** Under these primitives, no sequentially rational continuation supplies original-beneficiary wealth from B through an enforceable or discretionary onward payment. If the remaining hypotheses of Theorem 1 hold, its source bound therefore applies to sequentially rational feasible policies in this institutional subclass.

**Proof.** Primitives 1–2 exclude a recognized direct or onward obligation against B. At the last discretionary payment decision, a recipient can reduce a positive net outgoing payment and retain more cash without changing any completed benefit. Primitive 3 makes that deviation strictly beneficial. Backward induction eliminates earlier transfers whose sole return would be a later discretionary transfer. In an atomic net exchange benefiting N, some non-N participant must bear a net cash loss and may decline because no reciprocal benefit remains. Physically possible gifts therefore do not supply credible financing promises in this game.

Every earlier payment uses A. Claim prices and guarantee advances are internal transfers; summing dated budgets cancels them rather than counting price and face value twice. After initial investment  $k$ , later investment  $n - w$ , setup  $mF$  and required recovery  $mR$ , available balances are at most  $n - k + B_m + w$ . Add initial output  $ak$  and base receipts to obtain

$$\sum_i X_i \leq n - kd + B_m + w + \sum_i y_i b_i \leq n - kd + B_m + \bar{y}t.$$

The lost-output identity in Section 3.1 supplies the complementary bound  $\ell \geq gt$ . Pooling, fractional allocation, lotteries and transfers inside A do not defeat these bounds. QED.

The subclass does not exhaust the theorem's capability-defined comparison class. The rights and recourse exclusions remain substantive assumptions; the terminal argument explains why informal promises cannot substitute for them here. If B can be assigned to N, a binding assignment of  $1/20$  creates a payment path without creating cash. If a guarantee has effective recourse to B, it likewise changes the premise. Remaining valuable reciprocal trade can make informal forwarding credible. Additional accessible capital instead changes the inventory. None of these controls proves a fully funded economic alternative merely by removing this source certificate. A genuine common balance sheet supplying B-backed N wealth implements the excluded capability; calling it a pool, broker or separate application does not alter that conclusion.

## Appendix J. Voluntary formation and subsequent specialist entry

This separate date-0 problem compares complete funded continuations before finite public opportunities are known. A risk-neutral owner and incumbent choose baseline B or charter A with identical principal, priority and dated remedies; optional future fee rights can differ by their initial agreement. Specialist acceptance, funding, work and continuation costs are already included. Locally,  $G$  is owner expected gain before formation payments,  $r$  incumbent optional-fee loss,  $M$  real setup and  $W$  owner free cash after other obligations, idle under B.  $G$  and  $W$  differ from Theorem 1's notation.

Only nonnegative upfront owner-to-incumbent transfer  $x$  is allowed: no borrowing against future gains or reverse transfer. Funding and acceptance require

$$M + x \leq W, \quad x \geq r, \quad G - M - x > 0.$$

**Proposition J.1.** With  $L = \max\{0, r\}$ , voluntary funded adoption is possible exactly when

$$W \geq M + L \quad \text{and} \quad G > M + L.$$

**Proof.** Every transfer has  $x \geq L$ . The inequalities are necessary and, by choosing  $x = L$ , sufficient. All supporting transfers satisfy  $L \leq x \leq W - M$  and  $x < G - M$ . QED.

To compose this elementary participation test with Proposition 2, first verify each complete continuation's authorization, inherited rights, certification, entrant capital and service surplus. Include fees/costs once in  $G$  and  $r$ . Initial acceptance is risk-neutral here; later statewise protection uses Proposition 2's different benchmark and preference domain.

### Corollary J.2. Timely admission with arbitrary ordinary-fee sharing

Urgent and ordinary states have probabilities  $p, 1-p$ ,  $0 < p < 1$ . Certified output  $Y_j$  requires earlier work  $k_j \geq 0$ . In both arrangements the specialist owns  $K_e \geq \max\{k_u, k_o\}$  and receives  $k_j$  after output; unused funds stay with it. There are no other service/recovery costs. Set  $s_j = Y_j - k_j \geq 0$ . With the same inherited execution, fresh consent completes only ordinary work; the accepted charter completes both before their deadlines.  $M$  is paid at formation;  $W$  and allowed transfers are as in J.1.

Fresh consent pays the incumbent  $\beta s_o$ ,  $0 \leq \beta \leq 1$ , leaving owner  $(1-\beta)s_o$  in the ordinary state and zero urgently. The charter pays owner  $s_u$  or  $s_o$ , before  $M$  and  $x$ , and no optional consent fee. Zero-residual branches execute at indifference; they need not meet Proposition 2's strict-improvement condition. Put  $r = (1-p)\beta s_o$ . A strict owner gain with weak incumbent acceptance exists exactly when

$$ps_u > M, \quad W \geq M + (1-p)\beta s_o.$$

The supporting transfers and gains are

$$r \leq x \leq W - M, \quad x < r + ps_u - M.$$

$$\Delta_O = ps_u - M - (x - r), \quad \Delta_I = x - r, \quad \Delta_O + \Delta_I = ps_u - M.$$

**Proof.**  $G = ps_u + (1 - p)s_o - (1 - p)(1 - \beta)s_o = ps_u + r$ . Substitution into J.1 gives necessity and sufficiency, with  $x = r$  sufficient. Subtract setup and transfers for the gains. Both parties gain strictly exactly when  $ps_u > M$  and  $W > M + r$ : choose  $0 < x - r < ps_u - M$  with  $x - r \leq W - M - r$ . Specialist capital pays work before output, fees restore it, and the stipulated completion paths meet deadlines. No future receipt funds an earlier payment. QED.

Section 4.2 uses  $p = 1/2$ ,  $(Y_u, k_u) = (16, 4)$ ,  $(Y_o, k_o) = (6, 2)$ ,  $K_e = 4$ ,  $M = 1$ ,  $W = 7/2$ , total completion times one/two and deadlines one/three. For every  $\beta$ ,  $x = 2\beta + 1/2$  gives owner/incumbent gains  $9/2, 1/2$  and restores specialist wealth four. Expected additional cash surplus is five. Fee sharing changes distribution and formation liquidity, not that surplus. The cash gate is exact only in the stated transfer class; other funded credit or contingent compensation may change it.

For provider fee  $k_j + \mu_j$ ,  $\mu_j \geq 0$ , use  $s_j = Y_j - k_j - \mu_j \geq 0$ . Owner-incumbent gain remains  $ps_u - M$ ; adding the provider's urgent margin yields all-party cash gain  $p(Y_u - k_u) - M$ . At  $\mu_u = \mu_o = 1/10$ ,  $\beta = 1$ ,  $x = 5/2$ , owner/incumbent gains are  $22/5, 11/20$ . Margins are transfers, not resource consumption.

Against an alternative completing both services with identical outputs, specialist terms and inherited rights at date-0 real cost  $C_*$ , the charter's joint advantage is  $C_* - M$ . Equal capability/cost imply equality. Different timing, third-party claims, risk or funding require another complete comparison. Initial expected acceptance does not imply statewise improvement after sunk setup, nor does this conditional example predict market-wide unbundling.

### J.3. Proof and boundaries of the valuation bridge

Replace the formation parties' expected-wealth rule by common linear valuation  $V(z) = \lambda_u z_u + \lambda_o z_o$ , with positive weights summing to one. A sure unit at the dates used has value one; there is no uncounted carry, recovery or outside-opportunity cost. Physical probabilities remain positive. Actual payments still require dated cash, and this functional is not derived from Theorem 1's creditor utility.

Retain J.2's funded technology and specialist fees  $k_j + \mu_j$ ,  $\mu_j \geq 0$ , leaving  $s_j \geq 0$  and meeting its reservation wealth. Inherited terms, backing and feasible remedies are entry-independent; truthful observations and work are timely, including declared zero-residual execution at indifference. Let  $z_0$  be the owner's common inherited residual vector. Before formation payments,

$$\begin{aligned} Z_O^A &= z_0 + (s_u, s_o), & Z_O^B &= z_0 + (0, (1 - \beta)s_o), \\ Z_I^A &= (0, 0), & Z_I^B &= (0, \beta s_o). \end{aligned}$$

Thus owner gain is  $G_\lambda = \lambda_u s_u + \lambda_o \beta s_o$  and incumbent loss  $r_\lambda = \lambda_o \beta s_o$ . Formation requires

$$M + x \leq W, \quad x \geq r_\lambda, \quad G_\lambda - M - x > 0.$$

Since  $r_\lambda \geq 0$ , this intersection is nonempty exactly when  $W \geq M + r_\lambda$  and  $\lambda_u s_u > M$ ; choose  $x = r_\lambda$ . For every supporting transfer,

$$\Delta_O = \lambda_u s_u - M - (x - r_\lambda), \quad \Delta_I = x - r_\lambda, \quad \Delta_O + \Delta_I = \lambda_u s_u - M.$$

Strict gain to both replaces the cash inequality by  $W > M + r_\lambda$ . This proves Proposition 3's formation claim with J.2's unchanged funding and full-work deadlines.

For installed-position valuation before the state is known, an eligible buyer acquires precisely the displayed residuals, continuing funded service access and associated burdens, while creditors retain their actual claims. Exclude unrelated free cash; no setup or compensation remains due. Linearity gives

$$P_A^{\text{val}} - P_B^{\text{val}} = V(Z_O^A - Z_O^B) = G_\lambda.$$

These valuations are transaction prices only if the functional prices the relevant tradable entitlements consistently with access and replication. The creator's net gain is  $G_\lambda - M - x$ . Sale proceeds replace the future receipts sold, rather than supplementing them. Personal rights absent from the buyer's cash vector produce no inferred bare-share premium; wrapper transferability need not imply an issuer-wide premium. QED.

**Replication.** If both positions are tradable to the same parties and conversion at actual cost  $C$  creates the identical entitlement, preserving claims, consent, funding and timing without extra liability, a substituting purchaser will not pay more than  $P_B + C$ . With sufficient conversion capacity,  $P_A - P_B \leq C$ . A higher fixed-weight premium therefore requires adjustment of valuation, terms or access, or failure of a conversion premise. Capacity-limited formation can still create surplus without unlimited replication.

**Incidence.** New urgent output contributes  $\lambda_u s_u$  net of provider payments; ordinary fee transfer  $r_\lambda$  cancels between owner and incumbent. Add provider urgent margin  $\lambda_u \mu_u$  to obtain total cash valuation  $\lambda_u (Y_u - k_u) - M$ . This is not general welfare, lower margin or an issuer-wide price claim.

## Appendix K. Canonical stock illustration: domain and evidence

The local implementation uses one immutable, homogeneous share class with actual positions and exact-functioning cash transfers. The stockholder lends 100 units; a shop starts with 100 replacement units. Borrower and option buyer each start with 20,000 cash. The share-loan fee is 200, the option premium 300, replacement purchase 12,000 and strike 11,000. Successful settlement leaves cash balances 200 to the holder, 19,100 to the borrower, 8,700 to the buyer and 12,000 to the shop, with no cash in the controller: 40,000 total. The holder and buyer each finish with 100 shares. Replacement is bought before the escrowed strike is released. This is the exercised loss of 900 described in Section 5.2, not an ex-ante acceptance proof.

At call admission, expiry  $e$  plus the full one-day delivery window  $w$  must not exceed inherited loan due  $d$ . Exercise funding time  $t$  must satisfy  $t < e$ , hence  $t + w < d$ . Integer timestamps imply  $t + w \leq d - 1$ . At or after  $d$ , an unfilled exercise can therefore be refunded before original shares return; no timestamp simultaneously permits delivery and the expired-window refund. Reservation prevents replacement shares from being relended by this controller. If delivered, original and replacement positions transfer atomically. If unfilled, the buyer receives its strike back, retains no share-delivery guarantee and does not recover the premium under this product.

Assuming an accepted loan, the declared register/controller, exact cash transfers and transaction inclusion, the holder's return at or after  $d$  is already discharged or can be completed in at most two calls. With no active call, return directly. With an unaccepted or active expired call, expire it and return. With an unfilled pending exercise, refund the strike and return. With a settled call,

the replacement has already discharged the return claim. This is a conditional return remedy, not a wall-clock inclusion guarantee or unconditional option fulfillment.

Local tests reproduced return-blocking expiry and reservation defects, including an unaccepted expired offer that could not be terminated. The implementation described here admits only compatible new deadlines, expires unaccepted offers and prevents reservation reuse. Local test records support the bounded illustration; the timing, conservation and return arguments are given above. These tests concern a local model, not a production deployment.

Early recall, dividends, corporate actions, partial exercise, heterogeneous shares and issuer recognition are outside this implementation. The cash-additive extension of Proposition 2 does not prove those capabilities or this stock-delivery mechanism. Implementing them would require separate accepted terms, resources and compatibility arguments.

## Appendix L. A funded stock-lending equilibrium

This appendix uses its own local notation:  $n_s$  is physical share float and  $n_b$  is the fixed mass of eligible short sellers. It specializes the securities-lending equilibrium of Duffie, Gârleanu and Pedersen (2002, equations 3, 8 and 13–15), with homogeneous optimists and a per-completed-loan resource charge. The underlying pricing mechanism and its possible non-monotonicity are prior results. The additional construction specifies how accepted, funded authority changes executable opportunities while preserving inherited finance.

### L.1. Financial rights, information and resources

A registered share pays only cash redemption  $X$  at time  $T$ , with  $0 < x_{\min} \leq X \leq C$ . All agents agree that  $T$  has exponential intensity  $\gamma > 0$ , independent of  $X$  under their respective beliefs; no information about  $X$  arrives earlier. Optimists have  $E_H[X] = V_H$ , pessimists  $E_L[X] = V_L < V_H$ . There is no intermediate dividend, vote, recall or early delivery claim in this economy. Safe interest and separate collateral carry cost are zero in L.1–L.3; L.4 adds a positive liquidity-service opportunity cost.

An inherited bank advance  $0 \leq b \leq x_{\min}$  is due at  $T$ . The bank’s actual claim is  $b$ ; the holder’s residual is  $X - b$ . The accepted charter permits substitution of the payment backing under specified conditions. A short seller must first lock actual cash  $C$  and separately fund the agreed upfront lending fee and service charge. The share can then pass to the borrower for sale. At  $T$  the escrow pays  $b$  to the original bank,  $X - b$  to the original holder and  $C - X$  back to the borrower. Issuer cash  $X$  pays the current registered shareholder. These are distinct payment sources. Each later loan needs a new  $C$ ; the same reserve cannot support multiple closeouts. Thus the original bank’s payment, date and funded remedy are preserved state by state.

The contract is a binding cash-closeout loan: its deterministic fee is paid once at acceptance and there is no unilateral recall, cancellation or fee reset. The borrower’s net financial result is sale proceeds minus  $X$ , the fee and the service expenditure. This equals the financial payoff of delivering a borrowed share after buying it at its revealed redemption value. It does not satisfy an arbitrary obligation to return a share earlier or preserve interim voting rights. Cash-closeout claims are nontransferable replacement claims and cannot themselves be relended as registered shares.

Every subsequent registered buyer receives the same charter, bank-financing availability and full prospective lending-fee entitlement. The investor set is finite in mass, with more than  $n_s + n_b$  optimists and  $n_b$  eligible pessimists. Each investor is risk neutral and has a one-unit long or short

position limit; each pessimist can complete at most one short loan. Only the eligible pessimists may initiate short loans. Cash, registered shares, these bank advances and these loans are the admitted instruments. Separate free issuance or competitive trading of additional X-contingent claims is unavailable. The comparison holds eligible populations fixed and concerns two uniformly installed regimes.

For  $D = V_H - V_L - c > 0$ , sufficient common resources are  $C$  plus  $(1 + n_b/n_s)D + c$  per potential short seller, bank funds  $b(n_s + n_b)$ , and acquisition cash of at least  $V_H + n_b D/n_s - b$  per potential long purchaser. Initial implementation expenditure  $K$  also needs its own dated funds. These bounds follow from L.2. Unused balances remain owned by their suppliers. No future stock-sale proceeds prefund an earlier escrow. The construction counts actual capital; it does not claim a reduction in its quantity.

Raw contacts use the source model's continuum mass-action convention and intensity  $\bar{\lambda}$ . A short-lived opportunity requires authorization before expiry. In one charter fresh approval is available with probability  $a_0$ ; in the other a preaccepted funded substitution rule succeeds with probability  $a_1 \geq a_0$ . Approval marks are independent across contacts and independent of types, valuations and time. Failed contacts create neither a reserved share nor a continuing quote. Correct compatibility checks, execution before expiry and complete mediation of the escrow are premises. Independent thinning yields  $\lambda_j = a_j \bar{\lambda}$ .

The bank accepted its charter before reliance. Fresh consent is a nonstrategic availability friction here: no consent rent, additional bank operating burden or lost optional fee right is omitted from  $K$ . If any exists, it belongs in the formation comparison. An already capable institution belongs in the standing-rule regime. Neither the values of  $a_j$  nor this timing process are empirically estimated from the implementations studied here.

## L.2. Prices, participation and global verification

All values and pre-redemption dynamics in this subsection are conditional on  $T > t$ . Write  $k = \lambda n_s$ , let  $U(t)$  be unmatched short sellers and let  $S(t)$  be completed short loans, or short interest. A new loan replaces its lender's registered long position with a contractual long position, and transfers the same share to another buyer. Hence lendable physical float is  $n_s$ , while total long exposure is  $n_s + S(t)$ . Mass-action matching gives

$$U(t) = n_b e^{-kt}, \quad S(t) = n_b - U(t).$$

Let  $\ell(t)$  be prospective income from lending a newly purchased share,  $L(t)$  the agreed upfront fee at a contact, and  $B$  the searching borrower's value. A competitive provider receives  $c$  and consumes  $c$  once at completed execution. Let  $\eta \in (0, 1)$  be the lender's Nash bargaining share. With excess homogeneous optimists, a clearing price has  $P(t) = V_H + \ell(t)$ . Bargaining and the borrower Bellman equation then give

$$L - \ell = \eta(D - B), \quad \gamma B = k \max\{P - L - V_L - c - B, 0\},$$

$$B = \frac{(1 - \eta)kD}{\gamma + (1 - \eta)k}, \quad H = D - B = \frac{\gamma D}{\gamma + (1 - \eta)k} > 0.$$

The borrower's transaction payoff is  $B + (1 - \eta)H$ . For an unlent share, first-contact valuation gives

$$\ell(t) = \int_t^\infty e^{-\gamma(s-t) - \int_t^s \lambda U(v) dv} \lambda U(s) L(s) ds.$$

Consequently  $\ell' = \gamma\ell - \lambda U(L - \ell) = \gamma\ell - \lambda U\eta H$ . Its unique bounded solution is

$$\ell(t) = \frac{\eta \lambda U(t) H}{\gamma + k}, \quad L(t) = \ell(t) + \eta H, \quad P(t) = V_H + \ell(t).$$

The cancellation of the first-contact survival factor uses the lender's outside option; it does not allow one holder to lend an already-delivered share again. Successive registered buyers may lend, each with a separately funded borrower.

These values verify global decisions in the admitted market. The lender gains  $\eta H > 0$  above continuation, and the matched borrower gains  $(1 - \eta)H > 0$ . The bounded borrower Bellman solution dominates arbitrary rejection/waiting policies and stopping search at zero; when  $\lambda = 0$ , search and inactivity tie. The optimistic holder's value satisfies

$$P' + \gamma(V_H - P) + \lambda U \max\{V_H + L - P, 0\} = 0,$$

because  $P' = -k\ell$  and  $\lambda U\eta H = (\gamma + k)\ell$ . Selling yields  $P$ ; keeping and optimally lending has the same value. Purchase at any admissible stopping time has zero expected excess value, and excess indifferent optimists absorb every new short sale. This establishes a clearing allocation, not a unique allocation or strictly positive buyer surplus. A pessimist gains nothing by purchasing for lending:  $V_L + L - P = \eta H - (V_H - V_L) < 0$  at a contact and  $P' + \gamma(V_L - P) < 0$  between contacts. The accepted closeout is binding; an additional hedge requires actual cash. Bounded values and almost-sure finite T supply stopped-value verification without a terminal bubble.

The bounds  $\ell \leq n_b D / n_s$  and  $L \leq (1 + n_b / n_s) D$  justify the resources above. Banks receive  $b$ , providers break even after real expenditure, and lender/borrower acceptance is individually verified. The lump-sum interpretation is deliberate: the source paper's continuously renegotiated fee implementation is not imported after adding sunk  $c$ .

### L.3. Incidence, strictness and boundaries

Put  $d_j = (\gamma + n_s \lambda_j)(\gamma + (1 - \eta)n_s \lambda_j)$ . Direct subtraction gives

$$P_0(\lambda_1) - P_0(\lambda_0) = \frac{\eta n_b \gamma D (\lambda_1 - \lambda_0) [\gamma^2 - (1 - \eta) n_s^2 \lambda_0 \lambda_1]}{d_0 d_1}.$$

All denominator factors are positive, proving the stated strictness condition. Prospective lending income is maximized at  $\lambda^* = \gamma / [n_s \sqrt{1 - \eta}]$ . Meanwhile

$$E[S(T)] = \frac{n_b k}{\gamma + k}, \quad n_s \ell_0 + n_b B = DE[S(T)].$$

The last identity allocates subjective net trading gains between initial shareholders and potential short sellers after real service expenditure. It is not objective welfare when beliefs differ. Fees are

transfers,  $cE[S(T)]$  and  $K$  are real expenditure, and initial-owner adoption additionally requires funded  $K$  and  $n_s \Delta P > K$ . No forgone bank rent is being silently confiscated. A purchaser's price does not subtract a prior owner's already paid  $K$  again.

The Section 5.1 positive case is feasible with  $X \in \{60, 140\}$ , optimistic upper-state probability  $5/8$ , pessimistic probability  $19/40$ ,  $b = 50$  and  $C = 140$ . It has positive service expenditure and a strictly positive net owner gain after setup. The higher-intensity negative case has more completed loans and greater borrower value but lower owner capitalization. At intensities 1 and 2 the price increment is exactly zero. These exact examples and the separate-reserve accounting check support, rather than replace, the analytic proof.

Personal access, retained broker fees, heterogeneous partial adoption, a freely issuable derivative substitute, positive unmodeled carrying costs or a strategic incumbent's veto rent change the result. They are explicit boundaries of this particular equilibrium. The finite native-share loan/call prototype supplies evidence of bounded claim-compatible transitions; it does not implement this repeated cash-closeout economy or estimate its parameters.

#### L.4. Positive opportunity cost of net locked liquidity

The result need not rely on costless collateral. Retain L.1's admitted instruments, binding closeout, populations, funding and information, but let each eligible pessimist receive additive noncash utility at rate  $r > 0$  per unit of its own free cash until  $T$ . Escrowed cash supplies none; every free unit, including unused budgets and sale proceeds, supplies the same service. Other agents retain their preferences. This utility neither spends principal nor supplies new cash. Borrowing, fee payment and resale occur atomically in economic time, with actual prefunding in their logical order. Bank advances remain tied to protected share positions: resale repays the bank or preserves accepted backing. There is no unrestricted cash credit. A positive settlement interval or a positive market interest rate would require a different calculation.

Write  $\rho = r/\gamma$ , distinct from the authorization probability  $a$ . Relative to never trading, an executed borrower loses free cash  $h_C = C + L + c - P$  until  $T$ . Its gain is therefore  $A = (1 + \rho)(P - L - c) - V_L - \rho C$ . Sale proceeds restore services; charging an additional cost on the whole  $C$  would double count. Before a match the searcher keeps its original free balance, so subtracting that same no-trade baseline gives  $\gamma B = k \max\{A - B, 0\}$ . Define

$$D_\rho = V_H - c - \frac{V_L + \rho C}{1 + \rho} = \frac{D - \rho(C - V_H + c)}{1 + \rho}.$$

Write  $Q = E[S(T)] = n_b k / (\gamma + k)$  with  $k = \lambda n_s$ .

**Corollary L.4 (liquidity-service robustness).** If  $D_\rho > 0$ , the L.2 prices, fees and quantity path hold with  $D$  replaced by  $D_\rho$  and normalized borrower value  $\tilde{B}$ . Actual borrower utility is  $B = (1 + \rho)\tilde{B}$ . Proposition 4's finite price-change sign condition is unchanged. Owner net adoption remains  $n_s \Delta P - K$ , with actual funded  $K$ . The utility incidence is

$$h_C = C - V_H + c + \eta H > 0, \quad n_s \ell_0 + n_b B = [D - \rho h_C]Q, \quad H = D_\rho - \tilde{B}.$$

**Proof.** Divide the borrower's utility and disagreement value by  $1 + \rho$ . This positive normalization leaves the Nash maximizer unchanged. The bargaining product is proportional to  $(L - \ell)^n [P -$

$L - c - (V_L + \rho C)/(1 + \rho) - \tilde{B}]^{1-\eta}$ . Thus L.2's bargaining and Bellman system applies with  $D_\rho$ ; its bounded solution verifies accepting a match, refusing or waiting, and exiting. Optimists have unchanged preferences and retain the same holding, lending and selling verification. Their excess mass still clears the same physical and contractual long exposures. Matched gains are  $\eta H$  for the lender and  $(1 + \rho)(1 - \eta)H$  for the borrower. Here  $\eta$  is a Nash weight, not the lender's share of the sum of differently scaled utility gains.

A pessimistic financed purchaser's normalized valuation is  $v_b = (V_L + \rho b)/(1 + \rho) < V_H$ , since  $b \leq x_{\min} \leq V_L < V_H$ . Buying and later lending at equilibrium fees gives utility  $V_L + \rho b - (1 + \rho)V_H < 0$ . Between contacts,  $P' + \gamma(v_b - P) < 0$ . Even a different negotiated fee cannot make pessimistic ownership profitable: relative to immediate equity resale, a pessimistic lender and borrower have normalized combined match surplus  $-c - \rho(C - b)/(1 + \rho) - \tilde{B} \leq 0$ . Thus no fee gives both a gain; allowing the seller to resume short-search only raises its outside option. These tests and bounded-value stopping verification exclude buying, waiting for resale or relending as profitable deviations in the admitted market.

Since  $C \geq V_H$  and  $D_\rho \leq D$ , the original actual prefunding bounds remain sufficient; utility supplies no escrow funds. Substitution of  $P = V_H + \ell$  and  $L = \ell + \eta H$  gives  $h_C$  above. The normalized identity  $n_s \ell_0 + n_b \tilde{B} = D_\rho Q$ , together with  $n_s \ell_0 = \eta H Q$ , yields the stated actual incidence. Conditional on execution, remaining time has mean  $1/\gamma$ , so total expected foregone liquidity service is  $rh_C Q/\gamma$ . Real service expenditure is still  $cQ$ , and fees are financial transfers whose liquidity effect has already been counted. Differing beliefs and utility services make this a subjective-gains identity, not objective welfare. QED.

In the earlier positive example,  $r = 1/20$  and  $\gamma = 1$  give  $D_\rho = 8$ . The same increase from  $\lambda = 1/10$  to  $1/2$  yields  $\Delta P = 832/1155$  and, after  $K = 1/5$ , owner gain  $601/1155 > 0$ . At  $D_\rho < 0$ , no lending is an equilibrium; at equality, activity needs an indifference selection. This extension admits a positive, balance-consistent liquidity opportunity cost. It does not estimate that cost, reduce the required capital, or establish the result with unrestricted credit or arbitrary cash-flow timing.

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